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Transient and Steady-State Velocity of Domain Walls for a Complete Range of Drive Fields

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Abstract - Approximate analytic solutions for transient and steady-state 180° domain wall motion in bulk magnetic material are obtained from the dynamic torque equations with a Gilbert damping term. The results for the Walker region in which the transient solution approaches the familiar Walker steady-state solution are presented in a slightly new form for completeness. An analytic solution corresponding to larger drive fields predicts an oscillatory motion with an average value which decreases with drive field for reasonable values of the damping parameter. These results agree with those obtained by a computer solution of the torque equation and those obtained with the assumption of a very large anisotropy field.

INTRODUCTION

In a previous paper(1), transient solutions for the motion of domain walls in bulk magnetic materials were obtained from the vector equation of motion,

$$\frac{\partial \bar{M}}{\partial t} = -\gamma \bar{M} \times \left(\bar{H} - \frac{\alpha}{\gamma M} \frac{\partial \bar{M}}{\partial t}\right), \tag{1}$$

in which M is the saturation magnetization, γ is the gyromagnetic ratio, H is the effective field including applied, stray, anisotropy, and exchange components, and α is the damping parameter. With the coordinate system and wall configuration as represented in Fig. 1, the solutions obtained for θ, φ, and the corresponding wall velocity, v, are as follows

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$$\ln(\tan \theta/2) = C_2(t)\left[y - \int_0^t v(z)dz\right] \tag{2}$$

$$C_2(t) = \frac{M}{\mu_o} \sqrt{\frac{\mu_o}{2A}} \left(\sin^2 \varphi + h_k\right)^{1/2} \tag{3}$$

and

$$v = \gamma\sqrt{\frac{2A}{\mu_o}} \quad \frac{\alpha h_z + \sin \varphi \cos \varphi}{(1+\alpha^2)(\sin^2\varphi + h_k)^{1/2}} \quad , \tag{4}$$

in which φ is obtained from

$$(1 + \alpha^2) \frac{d\varphi}{dt} = \frac{\alpha}{2} \frac{\gamma M}{\mu_o} \left(\frac{2h_z}{\alpha} - \sin 2\varphi \right) . \tag{5}$$

In these equations A is the exchange constant, h_k is the normalized anisotropy field $\mu_o H_k/M$, h_z is the normalized applied field $\mu_o H_z/M$, and $\mu_o = 4\pi \times 10^{-7}$ mks. The only approximation involved concerns a consistency condition which requires that

$$\frac{dC_2}{dt} \left[y - \int_0^t v(\tau)d\tau\right] \ll C_2(t) \; v(t)$$

in order for $\varphi \neq f(y)$, a basic assumption in the trial solution. The solution under this assumption indicates that this approximation is an excellent one for the wall widths encountered in magnetic materials. This condition is equivalent to neglecting the incremental velocity along the wall compared to the velocity of the wall center. An alternate assumption which gives the same result concerns the partition of Eq. 1 such that the same terms equate to determine the structure constant, C_2, and the velocity at the wall center, v, during the transient as in the steady-state solution, which involves no approximation at least in the region $2h_z/\alpha \leq 1$.

For a stationary wall h_z, $\varphi = 0$ and the equations reduce to the familiar

$\ln(\tan \theta/2) = y\sqrt{K/A}$. For $2h_z/\alpha \leq 1$, the Walker solution[2] corresponding to $2h_z/\alpha = \sin 2\varphi$ predicts a contracted wall moving at constant velocity. The transient solution corresponding to this constant velocity solution is the one examined theoretically and experimentally in previous papers.[1],[3],[4] For completeness and for comparison, this solution is repeated in a slightly different form together with the solution for $2h_z/\alpha > 1$. All solutions assume a step function drive field, h_z.

Walker Region: $2h_z/\alpha < 1$

Eq. 5, which predicts that φ increases monotonically in time until it reaches a steady-state value corresponding to $2h_z/\alpha = \sin 2\varphi$, is integrated to yield

$$\tan \varphi = \frac{2h_z/\alpha}{1 + \sqrt{1-(2h_z/\alpha)^2}\, \coth \beta t} \tag{6}$$

with

$$\beta = \frac{\gamma M}{\mu_o} \frac{\alpha}{2} \sqrt{1 - (2h_z/\alpha)^2}\, /(1 + \alpha^2)$$

If the appropriate trigonometric functions of φ obtained from Eq. 6 are substituted in Eq. 4 and the hyperbolic function is expressed in terms of exponentials, the "exact" solution of reference (1) is obtained. Note that a factor in the effective damping β involves the drive field. With $2h_z/\alpha < 1$, the steady-state value of φ is less than $\pi/4$. Although φ itself is well-behaved, Eq. 4 is sufficiently complex that the transient response contains an overshoot for $2h_k^{1/4}(1 + h_k)^{1/4}[(1 + h_k)^{1/2}-h_k^{1/2}] < \frac{2h_z}{\alpha} < 1$, as previously shown.

Limiting Case: $2h_z/\alpha = 1$

In the limiting case, $\frac{2h_z}{\alpha} = 1$, either from Eq. 6 or directly from integration of Eq. 5,

$$\tan \varphi = \frac{(\gamma M/\mu_o)\alpha t/2}{(\gamma M/\mu_o)\alpha t/2 + (1 + \alpha^2)}$$

The angle φ increases monotonically to $\pi/4$ and the solution is similar to that of the previous case, with an overshoot in the transient response.

Oscillatory Region: $2h_z/\alpha > 1$

In this particular case Eq. 5 predicts that φ continues to increase for all time with a periodically changing rate. From Eq. 4 the velocity behaves as $\sin 2\varphi$ displaced by the relatively small term αh_z and modified by a positive term of oscillating magnitude in the denominator so that the wall velocity is alternately positive ($0 \gtrsim \varphi \lesssim \pi/2$) and negative ($\pi/2 \gtrsim \varphi \lesssim \pi$) and repeats. Again Eq. 5 may be integrated directly to obtain

$$\tan \varphi = \frac{2h_z/\alpha}{1 + \sqrt{(2h_z/\alpha)^2 - 1}\ \cot \omega t}$$

with

$$\omega = \frac{\gamma M}{\mu_o} \frac{\alpha}{2} \sqrt{(2h_z/\alpha)^2 - 1}\ /(1 + \alpha^2)$$

The periodic nature of the solution is apparent. The system is in a steady-state oscillatory condition from the beginning with no transient involved. The frequency depends on the drive field, h_z. In the limit of $2h_z/\alpha = 1$ the previous solution is obtained.

The maximum velocity magnitude in time is the same as the maximum steady-state velocity in the Walker region and corresponds to

$$\tan\varphi = \pm h_k^{1/4}/(1+h_k)^{1/4} ,$$

$$\sin\varphi\cos\varphi = \pm h_k^{1/4}(1+h_k)^{1/4}[(1+h_k)^{1/2} - h_k^{1/2}] ,$$

$$\sin^2\varphi = h_k^{1/2}[(1+h_k)^{1/2} - h_k^{1/2}] ,$$

and

$$|v_{max}| = \gamma\sqrt{\frac{2A}{\mu_o}}\,[(1+h_k)^{1/2} - h_k^{1/2}] ,$$

independent of the drive field, h_z. However, the time at which the maximum occurs depends on h_k and h_z and is given by

$$\cot\ \omega t = \frac{-1 \pm (2h_z/\alpha)(1+h_k)^{1/4}/h_k^{1/4}}{\sqrt{(2h_z/\alpha)^2 - 1}} .$$

Figs 2, 3, and 4 give the velocity as a function of time for several values of drive field and three values of h_k corresponding to permalloys and bubble-type materials.

For $\alpha^2(\frac{2h_z}{\alpha}) \ll 1$, the velocity is approximately equal to zero for $\varphi = 0,\pi$ which corresponds to $\omega t = 0, \pi$. The velocity is also approximately equal to zero for $\varphi = \pi/2$ which corresponds to

$$\cot\omega t\big|_{v=0} = -\frac{1}{\sqrt{(2h_z/\alpha)^2 - 1}} .$$

In general more time is spent in the positive velocity region and the velocity has a nonzero average value which is shown in Fig. 5.

In the limit of very large drives, $2h_z/\alpha \gg 1$, but again $\alpha^2(\frac{2h_z}{\alpha}) \ll 1$, the positive and negative half-cycles of velocty are an odd function about $\omega t = \pi/2$, $\varphi \cong \omega t \cong \gamma H_z t$ and the average velocity is zero. If in addition $h_k \gg 1$ the velocity behaves as sin $2\omega t$ at very high frequencies.

Conclusion

With suitable approximations a complete solution of the vector equation of motion with a viscous damping parameter may be obtained which describes the motion of a magnetic domain wall over a complete range of drive fields. The solution pertains to a bulk magnetic material described by an exchange constant, an anisotropy constant, a saturation magnetization, and a damping parameter. The oscillatory response predicted in the high drive region probably may best be explored experimentally in materials with relatively small saturation magnetizations and large anisotropy constants.

These results seem to agree with those reported by Slonczewski[(5)] and by Walker and Schryer[(6)] in a computer solution of the torque equation.

Acknowledgment

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Figure Captions

Fig. 1. Coordinate System and Wall Configuration

Fig. 2. Normalized Velocity as a Function of Time for $h_k = \mu_o H_k/M = 0.0005$ and Various Drive Fields $2h_z/\alpha = 2\mu_o H_z/\alpha M$

Fig. 3. Normalized Velocity as a Function of Time for $h_k = \mu_o H_k/M = 1.0$ and Various Drive Fields $2h_z/\alpha = 2\mu_o H_z/\alpha M$

Fig. 4. Normalized Velocity as a Function of Time for $h_k = \mu_o H_k/M = 5.0$ and Various Drive Fields $2h_z/\alpha = 2\mu_o H_z/\alpha M$

Fig. 5. Average Velocity as a Function of Drive Field for Various h_k in the Walker Region and in the Oscillatory Region

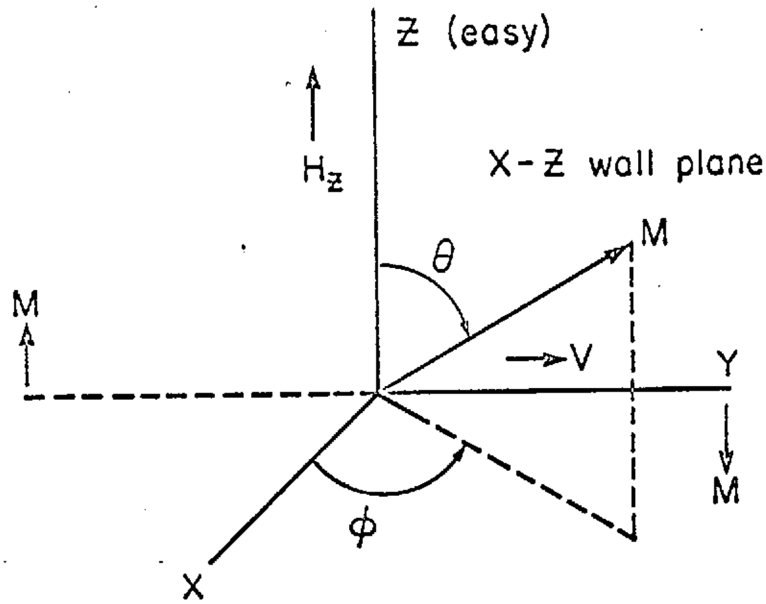

Figure 1
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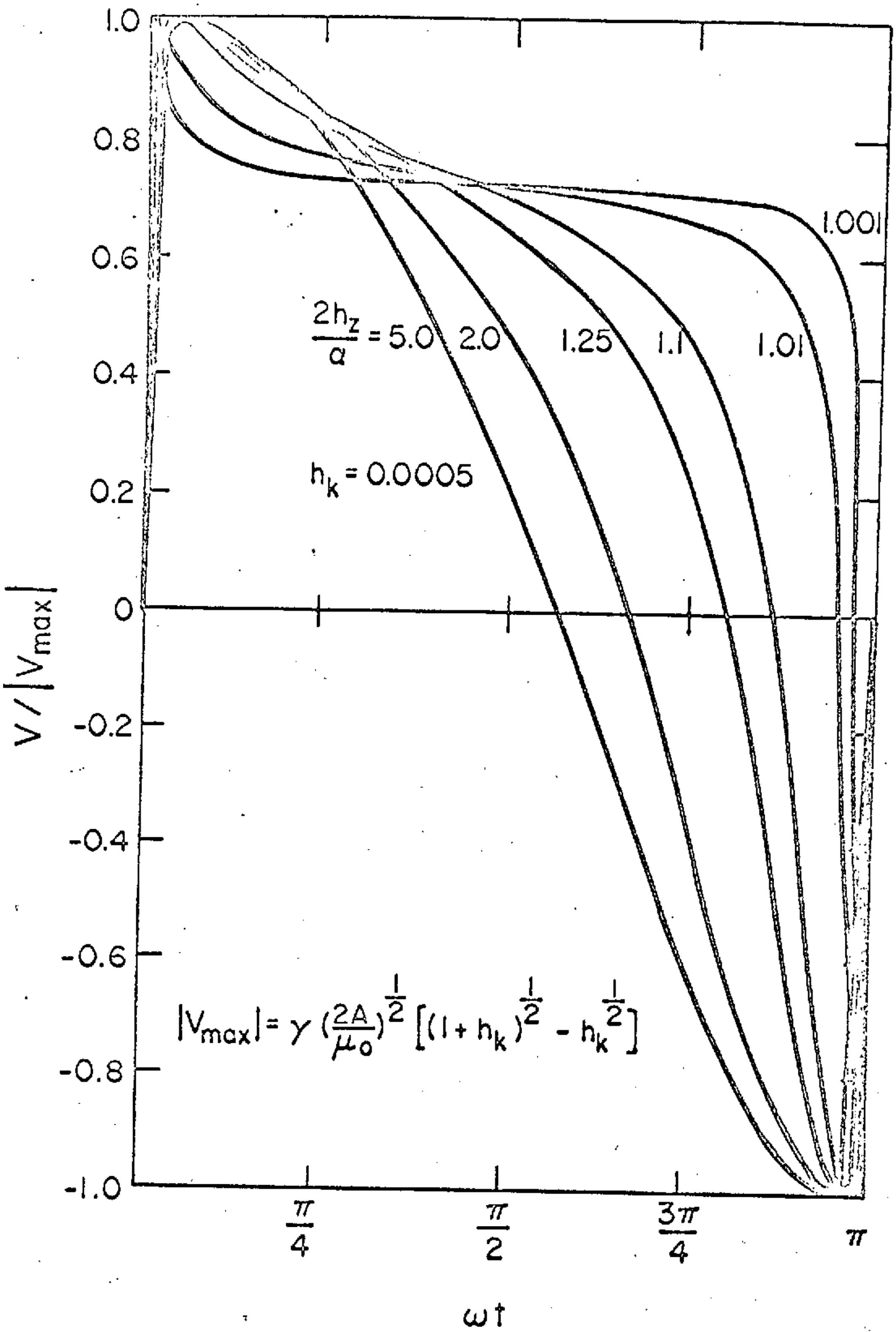

Figure 2
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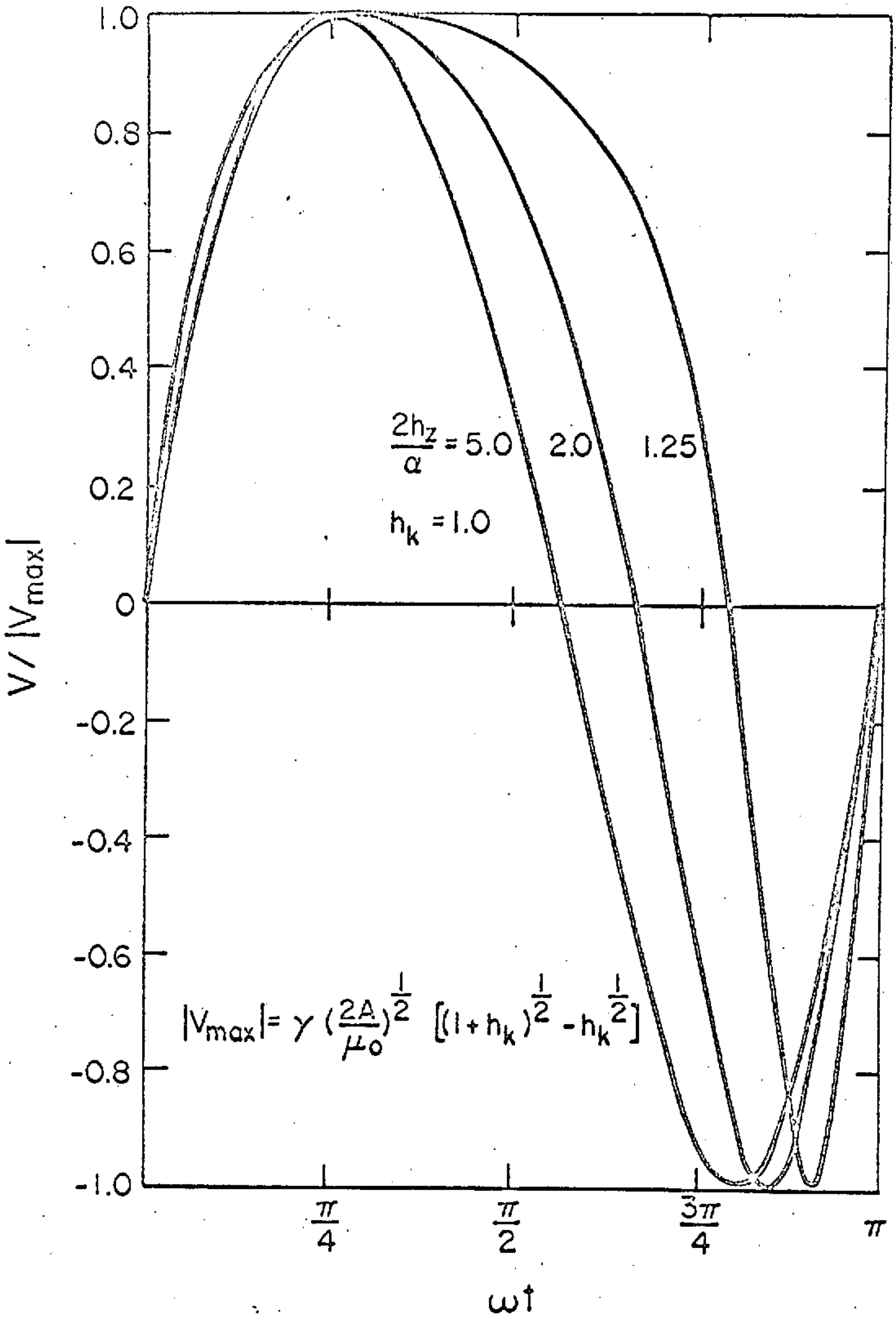

Figure 3
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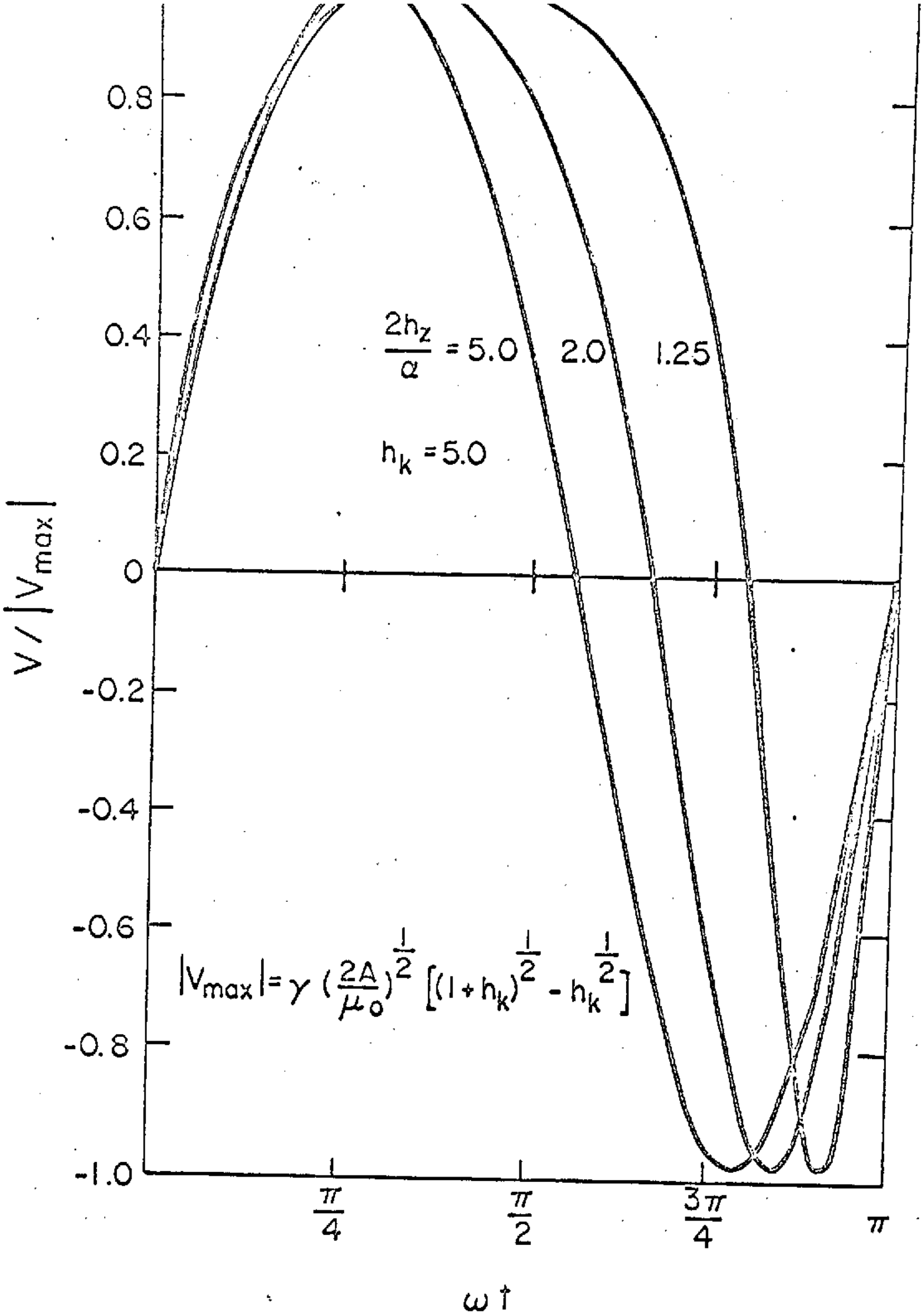

Figure 4
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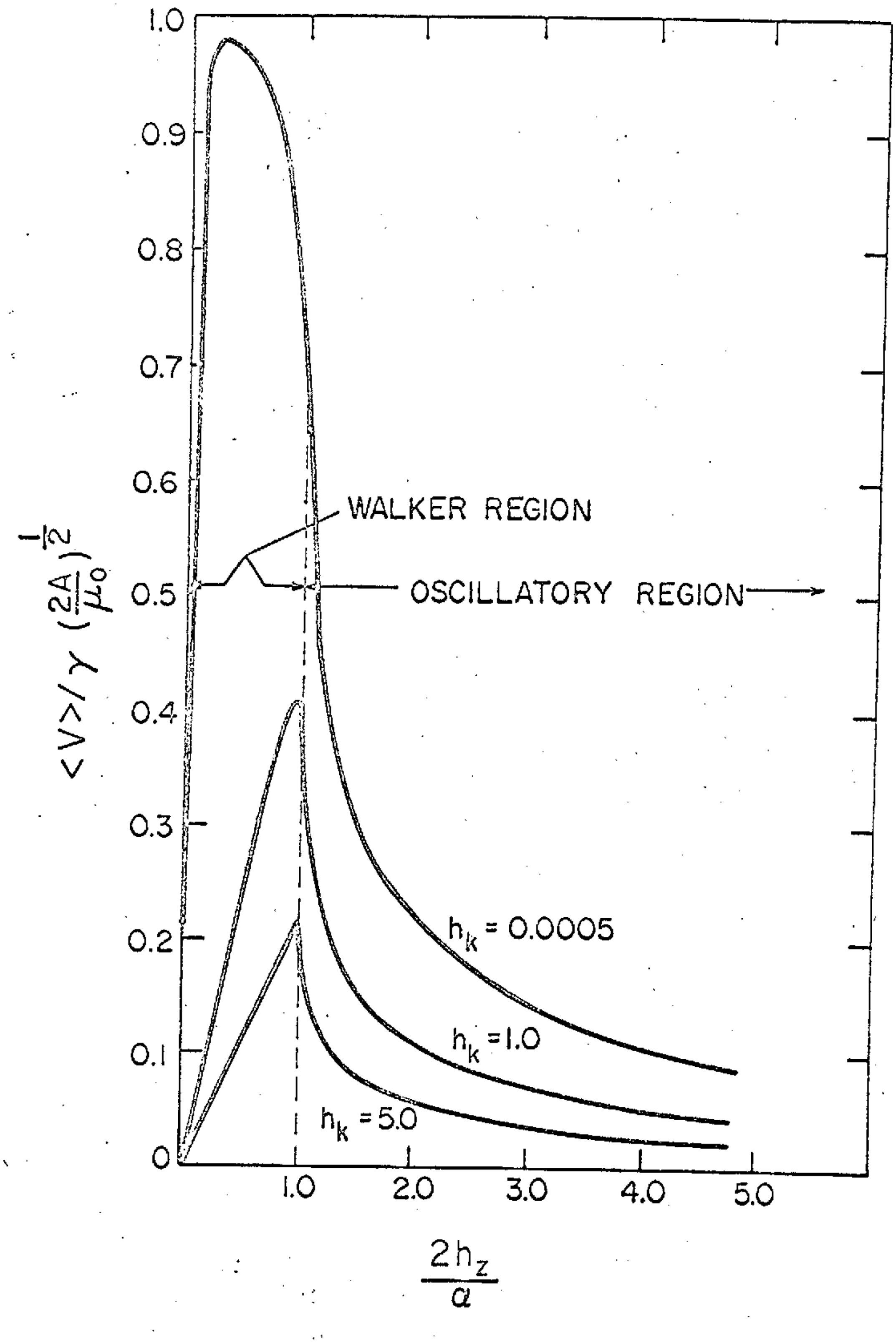

Figure 5
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